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Teacher Interventions in Fostering Students' Mathematical Understanding of Algebraic Expressions: An Analysis Based on Pirie Kieren Theory

Harfin Lanya[1,*], Susiswo[2], Moh Zayyadi[3], & Sharifah Osman[4]

[1,3]Teacher Professional Education Program, Universitas Madura, Indonesia

[2]Department of Mathematics Education , Universitas Negeri Malang, Indonesia

[4]Department of Science and Mathematics, Universiti Teknologi Malaysia, Malaysia

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Corresponding Author:

lanya_math@unira.ac.id

ABSTRACT

Teacher intervention is an action taken by teachers to guide students in reviewing their understanding. Comprehensive teacher intervention helps improve students' mathematical understanding by providing appropriate support throughout the learning process. This study aims to describe teacher intervention in mathematics learning on algebraic expressions. This study employed a descriptive qualitative research design involving eight mathematics teachers from eight junior high schools in Pamekasan during the preliminary observation. Based on the initial observations, three teachers were selected as the research participants. The study was conducted over three months. Data were collected through classroom observations, documentation, and transcripts of mathematics lessons. Data were analyzed using the Pirie–Kieren theory, with its eight layers of understanding serving as the analytical framework for examining teacher intervention. The findings show that the teachers primarily used convergent and probing follow-up questions to explore students' thinking. These interventions mainly supported students' understanding at the Structuring layer, where students began reorganizing their mathematical knowledge more systematically. However, interventions that facilitated students' progression to higher levels of understanding, particularly the Inventising layer, were not identified during the observed lessons. These findings indicate that teacher interventions remained focused on strengthening procedural and structural understanding rather than fostering students' ability to construct new mathematical ideas independently. Therefore, more varied, exploratory, reflective, and conceptually oriented interventions are needed to promote deeper mathematical understanding. The findings also highlight the importance of professional development programs that support teachers in designing learning environments that encourage reasoning, justification, and mathematical generalization, as well as sustained reflective teaching practices to continuously improve the quality of mathematics instruction.

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■ INTRODUCTION

Intervention is an action taken by teachers to direct students to review and develop their understanding during learning (Borgen, 2006; Gulkilik et al., 2020; Gülkılıka et al., 2015; Yao & Manouchehri, 2020). Comprehensive interventions that strengthen the learning process can improve students' understanding and achievement (Kahveci & Altun, 2019). In mathematics learning, teachers can intervene in various ways, particularly through questioning and scaffolding. Teacher questions encourage student responses, and those responses provide evidence of the depth of their understanding (Keeley, 2018; Parta, 2017; Walid, 2023). Scaffolding may involve environmental provisions, such as directing students to collaborate with peers; explaining, reviewing, and restructuring, such as providing simpler examples; and developing conceptual thinking through more complex examples (Julian, 2006; Manyuchi, 2016; Pfister et al., 2015). Thus, teacher interventions can take the form of questioning, facilitating collaboration, and providing examples that support students' mathematical understanding (Diyana & Sutopo, 2024).

Previous studies have examined teacher interventions in mathematics education (Bosman et al., 2021; Darmawan et al., 2021; Ellis et al., 2019; Effiyanti et al., 2023; Pirie & Kieren, 1994; Mustikaningtyas & Susiswo, 2020; Yao & Manouchehri, 2020; Norozpour, 2022). However, these studies have generally focused on deliberately planned interventions, whereas limited attention has been given to interventions that emerge naturally during classroom interaction. Such spontaneous interventions are important because they reflect teachers' real-time responses to students' mathematical thinking and may support or constrain the development of their understanding. Therefore, this study investigates the types of teacher interventions that naturally occur during the teaching of algebraic expressions within the Pirie–Kieren theoretical framework, examines their relationship to the layers of mathematical understanding, and identifies the layers where these interventions most frequently occur.

The Pirie–Kieren theory

The Pirie-Kieren theory conceptualizes mathematical understanding as a dynamic, non-linear, and recursive process in which students move forward and backward across layers of understanding through folding back, depending on their learning contexts and experiences (Pirie & Kieren, 1994). Although developed in 1994, the theory remains relevant because it focuses on the internal mechanisms through which students construct and develop mathematical understanding rather than on specific instructional models or intervention strategies.

In this study, the Pirie–Kieren theory serves as the sole analytical lens for examining how instructional interventions relate to the development of students' understanding at the cognitive level. Rather than conflicting with contemporary approaches such as game-based learning, scaffolding, or student-centered learning, the theory provides an interpretive framework for explaining how students interpret and internalize instructional practices and how these practices relate to the dynamics of their mathematical understanding.

Although the theory emphasizes non-linear and recursive development, the findings are presented sequentially by layers of understanding for analytical clarity. This sequential presentation does not imply a fixed or hierarchical progression. During coding, particular attention was given to folding back, whereby students returned to earlier layers to reconstruct or strengthen their understanding before moving forward. Such movements were coded as recursive connections between the current layer and preceding layers. Thus, the layers are treated as analytical categories rather than fixed developmental stages, and the analysis captures students' flexible and context-dependent movements across layers. The Pirie–Kieren framework consists of eight layers of mathematical understanding: Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Structuring, Observing, and Inventising (Gulkilik et al., 2020; Yao, 2020; Yao & Manouchehri, 2022). Accordingly, the sequential organization of the findings facilitates reporting and does not contradict the framework's fundamentally recursive nature.

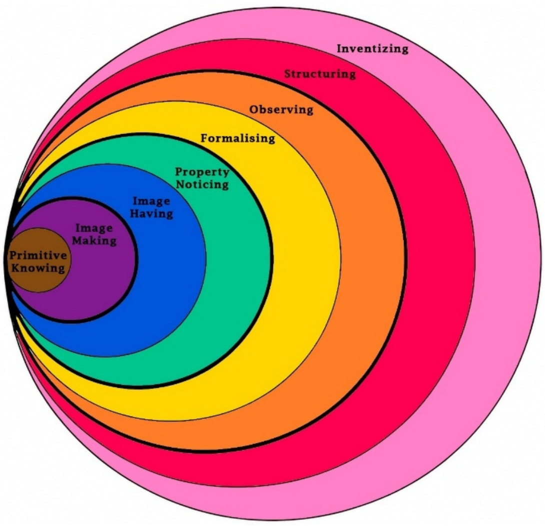

Figure 1. The Pirie-Kieren theory of mathematical understanding (Pirie & Kieren, 1994)

Figure 1 shows eight layers of a person's (student's) mathematical understanding. This theory serves as a lens for understanding and describing the growth of students' understanding (Rahayuningsih et al., 2022).

Types of questions

Questioning is an important form of teacher intervention in classroom learning. Previous studies have classified teacher questions according to their purposes, structures, and ways of responding to students. Moyer and Milewicz (2002), McCarthy et al. (2016), and Heinze and Erhard (2006) identify several forms of questioning, including probing and follow-up, leading questions, checklisting, and student-specific questioning. Heinze and Erhard (2006) further classify questions as reproductive, closed, open, evaluative, and rhetorical, while Moyer and Milewicz (2002) emphasize checklisting, teaching rather than judging, and asking questions followed by appropriate responses. Tofade et al. (2013) highlight the importance of waiting time, whereas Chin (2007) identifies repetition, reprocessing, and correction as forms of teacher feedback.

Questions may also be classified by instructional context. Aziza (2021) distinguishes conceptual, empirical, and value questions, while Davis (2003) differentiates convergent and divergent questions. Convergent questions generally focus on knowledge, comprehension, and application, whereas divergent questions encourage higher-order and critical thinking. Effective questioning, therefore, depends not only on selecting an appropriate question type but also on how teachers pose questions and respond to students' answers (Wilen, 2001; Wilen & Clegg, 1986). In particular, probing and follow-up questions can function as teacher responses to students' answers and provide opportunities to assess and extend their conceptual understanding (Barnes, 1976).

From the perspective of Bloom's taxonomy, questions range from knowledge and comprehension to application, analysis, synthesis, and evaluation (Bata Kristianus & Zamzani, 2020, 2021; Effendi, 2017; Lanya et al., 2024). Similarly, Paul and Elder (2007) categorize questions by goals and purposes: information, inferences, and conclusions; concepts and ideas; assumptions; implications and consequences; and viewpoints and perspectives.

Based on these perspectives, this study focuses specifically on convergent questions, probing and follow-up questions, and leading questions as forms of teacher intervention. These categories were selected because they directly capture how teachers guide, investigate, and extend students' responses during the learning process, particularly in relation to developing mathematical understanding.

■ METHOD

Research Design

This research is a qualitative descriptive study in which the researcher is the main instrument (Creswell, 2012). The data source is learning videos produced by three junior high school mathematics teachers while teaching students algebraic expressions, with 2 learning videos each. The learning videos are then transcribed and analyzed using the N-Vivo application. Based on the learning video transcripts, especially the introductory, core, and closing activities, the researcher focuses on the teacher's intervention in students' mathematical understanding.

Participant

The participants in this study were mathematics teachers from SMPN 1 Pamekasan and SMPN 4 Pamekasan who fulfilled the predetermined inclusion criteria established by the researchers. A purposive sampling strategy was adopted because the study sought participants with demonstrated experience in implementing interactive learning practices in mathematics instruction. Participant selection was carried out through a three-stage screening process before the primary data collection. The first stage involved examining teachers' lesson plans to identify evidence of student-centered instructional approaches, classroom discussions, collaborative learning activities, and opportunities for active student engagement. The second stage consisted of preliminary classroom observations using a structured observation guide developed from established characteristics of interactive learning described in the literature. The guide assessed five key aspects: reciprocal teacher–student communication, opportunities for students to ask and respond to questions, teacher facilitation of classroom discussions, collaborative problem-solving activities, and active student participation throughout the learning process. In the final stage, semi-structured interviews were conducted to verify that these instructional practices were routinely implemented rather than occurring only during the observed sessions. As this research employed a qualitative design, participant selection did not rely on predetermined quantitative scores. Instead, teachers were selected based on the consistent demonstration of the identified interactive learning indicators during the preliminary observations. The observation results were subsequently cross-checked with lesson plan analyses and interview findings to ensure the credibility and consistency of the selection process through data triangulation. After this screening, three mathematics teachers were purposively selected as participants. All selected teachers taught Grade VII mathematics, with a particular focus on algebraic expressions.

Data Collection Tools

Data were collected through classroom documentation using video recordings captured by cameras and mobile devices to record teaching and learning activities. The recorded videos were subsequently transcribed and used as primary data for analysis. To ensure methodological clarity, each instrument was defined using explicit indicators. The observation instrument focused on identifying specific teacher and student behaviors during learning. Teacher behaviors included the types of instructional interventions provided, questioning strategies, feedback patterns, and facilitation of student discussions. Student behaviors included responses to teacher interventions, participation in discussions, problem-solving approaches, and shifts in levels of mathematical understanding.

The interview instrument explored the teacher's reasoning behind instructional interventions. Key questions addressed (1) the teacher's intentions when posing particular questions, (2) considerations in responding to students' difficulties, (3) reasons for choosing certain forms of assistance or prompts, and (4) reflections on how their interventions supported students' mathematical understanding.

Data collection was also conducted through semi structured interviews to gain deeper insight into the teachers' instructional decisions and reflections. To ensure the trustworthiness of the qualitative data, the study employed several validation strategies. First, data triangulation was carried out by comparing findings from classroom observations, video transcripts, and interview data to confirm the consistency of identified instructional interventions. Second, peer debriefing involved discussions with fellow researchers to review coding decisions and interpretations of teacher interventions. Third, member checking was implemented by sharing selected interview transcripts and preliminary interpretations with participating teachers to verify the accuracy of the researchers' understanding of their instructional practices. These procedures enhanced the credibility and reliability of the study's findings. The interviewer conducted an interview with the teacher while playing back the video of the teacher intervening in learning and asked questions to ascertain the reasons for and impact of the teacher's intervention on students. Meanwhile, the interview conversations were recorded using a cell phone. Next, the transcript results were analyzed using the N-vivo application.

Data Analysis

The data were analyzed using a qualitative data analysis approach following the interactive model proposed by Creswell and Creswell (2023). The analysis consisted of four interconnected stages: data collection, data condensation, data display, and conclusion drawing and verification. NVivo 14 software was used to facilitate the organization, coding, and retrieval of qualitative data.

The analysis focused on each teacher intervention that occurred during mathematics learning activities on algebraic expressions. Primary data included classroom observation transcripts and video recordings of three junior high school mathematics teachers. During the coding process, all transcripts and videos were repeatedly reviewed to identify meaningful segments representing teacher interventions.

Coding was conducted in two stages. First, open coding identified and labeled all forms of teacher interventions in the classroom interactions. Next, axial coding grouped similar codes into broader categories based on the characteristics and functions of the interventions. Throughout the analysis, codes identified in the transcripts were continuously compared with the video recordings to ensure consistency and preserve the contextual meaning of each interaction. Finally, the categorized data were interpreted to identify patterns of teacher interventions in interactive mathematics learning. The analysis focused on each intervention the teacher used during learning activities. In the analysis, researchers reviewed transcripts and videos of junior high school mathematics teachers teaching algebraic expressions.

The researchers observed a relationship between teacher movements or interventions and students' layers of mathematical understanding, as well as the types of interventions that emerged during students' mathematical understanding activities. This relationship was identified through a qualitative, coding-based analysis systematically developed and analyzed using NVivo.

The coding framework used a deductive–inductive approach. Deductively, the main coding categories were derived from the layers of understanding in the Pirie–Kieren theory, which served as a conceptual framework for identifying characteristics of students' mathematical understanding activities. Inductively, subcategories were generated from empirical data through an open coding process applied to transcripts of classroom interactions, allowing for the emergence of context-specific forms of teacher intervention that were not fully anticipated by the initial theoretical framework. Table 1 presents the NVivo 14 coding scheme for teacher intervention in classroom instruction.

To ensure consistency and reliability, the analysis was conducted iteratively on transcripts from three instructional sessions for each research participant. Based on the

Table 1. Nvivo 14 coding scheme for teacher intervention

Parent Node	Open Coding	Axial Coding	Frequency	Definition	Indicator
Teacher Intervention	Probing and follow-up	Primitive Knowing	44	Make an effort to understand new definitions	Define variables, coefficients, terms, and constants
	Convergent question				
	Probing and follow-up	Image Making	19	Make a conceptual overview through pictures and examples	Distinguish variables, coefficients, and constants in algebraic expressions
	Convergent question				
	Small group discussion	Image Having	20	Create a mental picture through the topic without having to work through examples	Create an example of an algebraic expression
	Probing and follow-up	Property Noticing	21	Recognizing the similarities and differences of the various descriptions of a topic and developing them into a concept definition built between these images	Grouping similar variables, coefficients, and constants
	Convergent question				
	Probing and follow-up	Formalizing	19	Make an abstraction of a mathematical concept based on the properties that appear	Defines an algebraic expression
	Convergent question				
	Convergent question	Observing	6	Enable coordinate formal activities at the previous level in order to be able to use it on related issues	Operates on algebraic expressions
	Leading question			Enable to make a formal statement about a mathematical concept	
	Giving example				
	Probing and follow-up				

	Convergent question	Structuring	12	Enable linking the relationship between a theorem and other theorems and be able to prove it based on logical arguments	Explain the terms of the two algebraic expressions: additive and subtractable
	Leading question				
	Giving example				
	Probing and follow-up				
		Inventising	0	Able to create new statements that can grow into a new concept	Make a new statement about the algebraic expression. For example, not all variables can be added or subtracted; only similar variables can be added or subtracted

independent analyses, the data were coded using a coding scheme aligned with the layers of understanding in the Pirie–Kieren theory. The coding process yielded a Percentage Agreement of 82.35%, indicating strong intercoder agreement. Any discrepancies in the coding were subsequently discussed and resolved through consensus by referring to the operational definitions of each category and the theoretical framework of teacher intervention. This procedure minimized subjective interpretation and enhanced the consistency, dependability, and credibility of the coding process.

To address potential ambiguity when a single teacher intervention could fit more than one layer of understanding, the researchers avoided imposing a rigid, singular classification. Instead, categorization was based on the intervention's dominant function within the ongoing instructional interaction, taking into account students' immediate responses and the direction of their developing understanding. This approach aligns with the non-linear and recursive nature of the Pirie–Kieren theory, which recognizes that a single intervention may trigger complex, dynamic shifts in understanding.

After completing the coding process, the coded data were organized and examined in NVivo 14 to identify relationships between teacher intervention types and the dynamics of students' layers of mathematical understanding. The coding results were used to classify each teacher intervention according to the predefined analytical categories and to examine how these interventions corresponded to different layers of students' mathematical understanding.

■ RESULTS AND DISCUSSION

The results revealed that each subject delivered distinct forms of intervention across the layers of understanding. Among the 3 subjects, only 2 intervened up to the "Structuring" layer of understanding, while the other subject intervened only up to the "formalizing" stage. In each layer of understanding, subjects provide different treatments; the following explains each subject's intervention.

IY Subject and His Learning

Primitive knowing

At this layer, the teacher provided an intervention in the form of convergent questioning (*The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix A.1*). The convergent questions were posed to identify students' prior knowledge and initial understanding of algebraic expressions. Based on the indicators of understanding in the Pirie–Kieren theory, at this layer students could identify and explain basic components of algebraic expressions, including variables, coefficients, terms, and constants.

In the Pirie–Kieren theory, understanding begins with Primitive Knowing, which

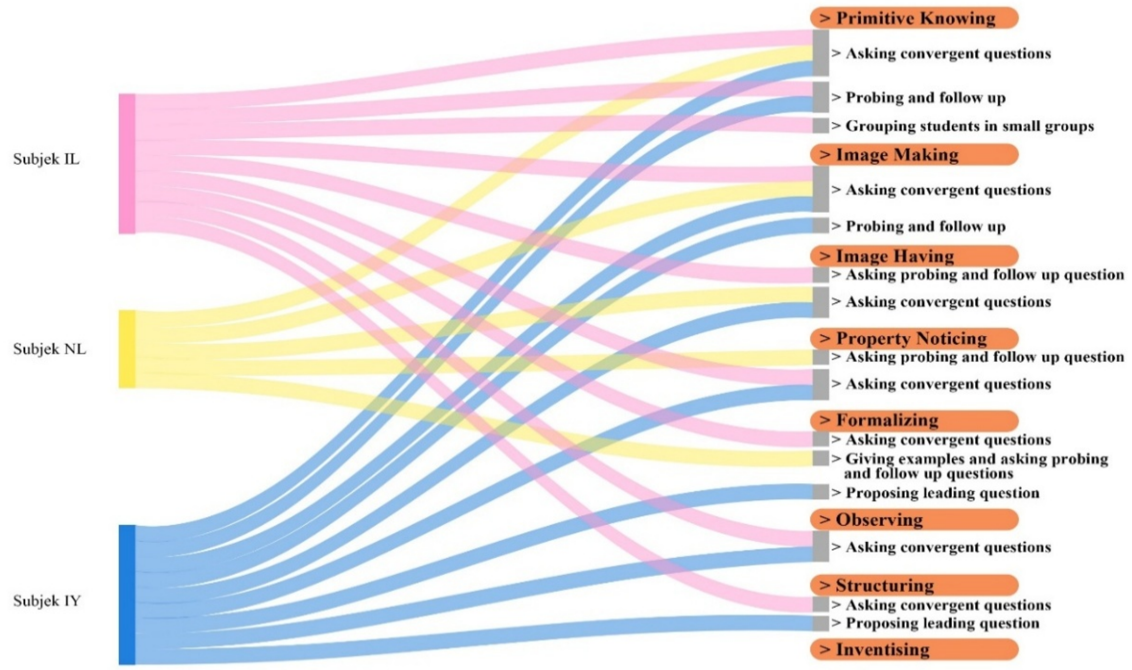

Figure 2. Analysis for 3 subject

represents the knowledge available to learners as a starting point for developing further mathematical understanding (Yao & Manouchehri, 2020b). In this study, the evidence from the teacher–student interaction and the follow-up interview indicates (Appendix 1.a) that students entered the learning activity with some accessible knowledge of algebraic expressions. The teacher's convergent questioning elicited this knowledge by directing students' attention to familiar components of an algebraic expression, namely variables, coefficients, constants, and terms. Therefore, the interaction provides evidence that students' understanding at the Primitive Knowing layer involved activating and articulating previously acquired knowledge about basic algebraic concepts.

Image making

At this layer, the teacher provided interventions in the form of convergent, probing, and follow-up questions (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix A.2*). These questions were used to direct students' attention to the specific components of an algebraic expression and to encourage them to clarify and justify their responses. The interaction shows that students initially responded by referring to "same and non-similar terms," which did not directly answer the teacher's question. Through follow-up and probing questions, the teacher redirected students' attention to the first algebraic expression and asked them to determine the number of terms. The students subsequently identified that the expression consisted of two terms.

According to the Pirie–Kieren theory, students at the Image Making layer engage in specific actions through which they construct initial images of a mathematical concept. These conceptual images remain closely connected to the actions and representations that form them (Yao & Manouchehri, 2020a). In this study, students identified the number of terms by directly examining the given algebraic expression. The teacher's convergent questions, together with probing and follow-up questions, directed students to examine the expression, identify its components, reconsider their initial responses, and articulate the number of terms. Thus, the interaction provides evidence of students' emerging conceptual images of algebraic expressions at the Image Making layer.

Image having

At this layer, the intervention involves asking students convergent questions. According to the Pirie–Kieren theory, at the Image Having layer, students can hold or recall an image of a mathematical concept without necessarily re-engaging in the actions that initially constructed it. In this study, students' ability to spontaneously provide examples such as “3a” and “4C” suggests that they could access an internal representation of an algebraic expression and express it symbolically. Thus, the teacher's convergent questioning functioned as a means of eliciting students' existing conceptual images, while the students' independent responses provided evidence that their understanding had progressed from constructing an image through activity toward having an accessible mental image of an algebraic expression (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix A.3*).

Property noticing

In this layer, the teacher intervened by asking students convergent questions. In the Pirie–Kieren theory, the Property Noticing layer involves students becoming aware of and identifying specific properties of a mathematical concept based on images and mathematical objects they have previously encountered. At this layer, students move beyond simply holding an overall image of a concept and begin to identify particular characteristics that can be used to describe or reason about it. In this study, the students' identification of p and q as variables indicates that they had begun to attend to the relationship between the symbols in the expression and their mathematical role. The teacher's convergent questioning supported this process by directing students' attention toward the variable as a specific property of the algebraic expression. Therefore, the interaction and the teacher interview provide evidence of students' understanding at the Property Noticing layer (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix A.4*).

Formalizing

In this layer, the interventions involved providing examples and asking leading questions. Based on the Pirie–Kieren framework, at this layer students began to formalize their understanding of adding and subtracting algebraic expressions by connecting their intuitive understanding of similarity with the formal concept of like and unlike terms. This interaction indicates the Formalizing layer, in which students begin to articulate mathematical ideas using more formal language, properties, and relationships. In this study, students' understanding progressed from the intuitive recognition that “the same” objects can be combined toward the mathematical principle that algebraic terms can be combined when they share the required characteristics. The teacher's concrete examples and leading questions supported this transition by connecting students' intuitive reasoning to the formal mathematical notion of like terms. Thus, the interaction and the teacher interview provide evidence that students were beginning to formalize their understanding of adding and subtracting algebraic terms (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *A.5*).

Observing

At this layer, the intervention involved providing examples and asking convergent questions. Students begin to observe and reflect on the effects of mathematical operations and the relationships among previously established properties (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix A.6*). In this study, the student applied the previously established idea that quantities of the same type can be combined and then carried out the corresponding addition. Therefore, the intervention through concrete examples and convergent questioning allowed the student to enact and observe combining like quantities, supporting development at the Observing layer.

Structuring

In this layer, the intervention involved giving examples and asking leading questions. This interaction reflects the Structuring layer because students began integrating previously noticed properties and procedures into a coherent understanding of algebraic operations. The teacher's examples and leading questions served as an intervention that helped students organize these relationships and articulate the prerequisite for adding or subtracting algebraic terms. Specifically, students were guided to identify like terms, group them, and then simplify by operating on their coefficients. The teacher's interview confirms that this sequence was an intentional instructional strategy to help students connect these mathematical ideas. Therefore, the interaction and the teacher interview provide converging evidence that students developed a more structured understanding of algebraic operations at this layer (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *A.7*).

Inventising

Across all classroom observations, no evidence was found that students reached the Inventising layer of the Pirie-Kieren framework, nor were instructional interventions observed that explicitly facilitated learning at this level. Rather than reflecting the instructional practice of a particular teacher, this consistent pattern across the observed classrooms suggests that opportunities for students to generate new mathematical ideas, formulate original conjectures, or extend existing concepts beyond the given tasks were limited.

One possible explanation is that classroom instruction focused primarily on developing procedural fluency and ensuring mastery of the intended curriculum outcomes for algebraic expressions. Most learning activities emphasized solving structured problems with predetermined solution strategies, leaving limited opportunities for exploratory, open-ended, or inquiry-based mathematical tasks that typically support the Inventising layer. In addition, instructional time constraints and the need to complete curriculum targets may have encouraged teachers to prioritize students' attainment of foundational conceptual understanding before engaging them in more advanced mathematical exploration.

These findings indicate that the absence of the Inventising layer should not be interpreted as a deficiency of individual teachers, but rather as a characteristic of the instructional context observed in this study. They highlight a potential gap between classroom practices and the development of higher-order mathematical thinking envisioned by the Pirie-Kieren framework. Future research should therefore investigate instructional designs that intentionally incorporate open-ended investigations, mathematical inquiry, and opportunities for students to formulate and justify original mathematical ideas in order to support progression toward the Inventising layer.

IL Subjects and His Learning

Primitive knowing

At this layer, the intervention provided by involved grouping students into small groups and activating their previously acquired mathematical knowledge before introducing the algebraic material. Primitive knowing refers to the knowledge students have before and at the beginning of developing a particular mathematical understanding. It serves as a starting point from which students can construct and reorganize subsequent layers of understanding (Yao & Manouchehri, 2020b). In this study, students' prior knowledge of integer addition and subtraction constituted an important starting point for learning algebraic operations. Subject IL's intervention through recalling prerequisite material, organizing students into small groups, and providing a concrete pattern activity enabled students to access and articulate this existing knowledge (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix B.1*).

Image making

At this layer, the intervention involved asking students convergent questions. From the perspective of the Pirie–Kieren framework, the Image Making layer refers to the process through which students construct an initial image of a mathematical concept through specific actions involving mathematical representations. At this layer, students' understanding remains closely tied to the objects, representations, or actions through which they construct the image. In this study, students were asked to examine a particular algebraic expression, identify the parts separated by addition or subtraction, and determine the number of terms. These actions provided students with an opportunity to construct an initial image of the structure of an algebraic expression. The teacher's convergent and follow-up questions played an important role in this process. The questions directed students' attention to specific features of the displayed expression and encouraged them to reconsider their responses based on what they could observe in the representation. Therefore, the evidence suggests that students were developing an initial image of an algebraic expression by identifying and counting its terms. This interpretation aligns with the Image Making layer of the Pirie–Kieren framework because students' understanding remained closely tied to the specific algebraic representation and the actions performed on it (The learning activity transcripts and researcher–teacher interview transcripts are provided in *Appendix B.2*).

Image having

In this layer, the intervention carried out by asking probing and follow-up questions to students (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix B.3*). From the perspective of the Pirie–Kieren framework, this interaction provides evidence of the Image having layer because students were able to extend their existing understanding of algebraic expressions to formulate a general relationship between quantities. The interaction indicates that students not only recognized or operated on algebraic expressions but also began to generate and generalize an algebraic relationship independently.

Property noticing

At this layer, the intervention involved asking students convergent questions. Based on the Pirie–Kieren framework, students at this layer began to notice and articulate specific properties of algebraic objects, particularly the distinction between similar and different variables and the conditions under which quantities can be considered similar. From the Pirie–Kieren framework perspective, the interaction reflects the Property Noticing layer. At this layer, students begin to identify and attend to particular properties of a mathematical object rather than merely recognizing or representing the object as a whole. The teacher's convergent questioning supported this process by progressively directing students' attention toward the relevant property. First, students distinguished between a marker and an eraser. Second, the teacher asked students to explain their reasoning. Third, the teacher transferred the same distinction to the algebraic symbols (x) and (y). This sequence enabled students to identify a particular characteristic and recognize its relevance in an algebraic context (*learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix B.4*).

Formalizing

In this layer, the intervention involved asking students convergent questions. Based on the Pirie–Kieren framework, at the Formalizing layer, students begin to express their mathematical understanding using formal mathematical language, symbols, and established properties. In this study, students were guided to formalize their understanding of the condition for adding algebraic terms. This interaction reflects the Formalizing layer because students connected their emerging intuitive understanding to formal mathematical language and established mathematical properties. At this layer, the learner begins to express and organize understanding using conventional mathematical terminology and representations (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix B.5*).

Observing

At this layer, the intervention involved asking students convergent questions. Based on the Pirie–Kieren framework, at the Observing layer, students use previously established mathematical properties to examine and operate on mathematical expressions. In this study, students were guided to apply their understanding of like and unlike terms when determining whether two algebraic terms could be combined. This interaction reflects the Observing layer because the student used the previously established concept of like and unlike terms when operating on an algebraic expression. The teacher's convergent questions supported the student in examining the terms' symbolic form and determining whether the operation could be performed. Therefore, the interaction indicates that the student applied the structural property of algebraic terms in an operational context. However, the student's explanation remained relatively informal (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *B.6*).

Structuring

At the Structuring layer, the intervention carried out involved asking convergent questions that encouraged students to connect previously developed mathematical ideas and organize them into a more coherent relationship. Based on the Pirie–Kieren framework, Structuring involves students making connections among previously developed images, properties, and mathematical relationships so that their understanding becomes more organized and integrated. This interaction reflects the Structuring layer because the student connected previously developed knowledge about multiplying algebraic terms with the commutative property of multiplication. The teacher's convergent questions helped students organize these ideas into a coherent relationship between the order of factors and the resulting algebraic expression. Therefore, the interaction indicates that students were beginning to understand not only how to multiply algebraic expressions but also why equivalent results can be obtained when factors are rearranged (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *B.7*).

NL subjects and His Learning

Primitive knowing

At the Primitive Knowing layer, the intervention provided involved asking convergent questions to elicit and activate students' prior mathematical knowledge (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix C.1*). Based on the Pirie–Kieren framework, Primitive Knowing refers to the knowledge that learners already possess and can draw upon as a basis for developing further mathematical understanding. At this layer, students are not necessarily expected to demonstrate a fully formal understanding of a concept; rather, they begin by recalling, recognizing, and expressing ideas relevant to the mathematical concept being introduced. This interaction reflects the Primitive Knowing layer because the students' responses draw on knowledge accessible before, or at the beginning of, the formal development of the algebraic concept. The students could recognize symbols, represent repeated quantities, and connect familiar objects with simple algebraic notation. These forms of knowledge function as resources for subsequent development of mathematical understanding.

Image making

At the Image Making layer, the intervention provided involved asking convergent questions to help students construct an initial image of the components of an algebraic expression. Based on the Pirie–Kieren framework, Image Making refers to the stage in which students develop an initial understanding of a mathematical concept through interaction with representations, examples, and mathematical activities. This interaction demonstrates how the teacher's questioning created an opportunity for students to refine an informal representation into an emerging mathematical understanding, which can subsequently support movement toward more formal mathematical knowledge in later layers of the Pirie–Kieren framework (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix C.2*).

Image having

At the Image Having layer, the intervention carried out involved asking convergent questions that encouraged students to access and use an existing image of algebraic representation in a new situation. Based on the Pirie–Kieren framework, Image Having refers to the stage at which students can work with an image of a mathematical idea that has already been developed, without needing to reconstruct the original activity each time (*The learning activity transcripts and researcher–teacher interview transcripts are provided in the appendix C.3*).

Property noticing

At the Property Noticing layer, the interventions carried out involved providing examples and asking probing and follow-up questions to direct students' attention to particular properties and relationships within an algebraic situation. Based on the Pirie–Kieren framework, Property Noticing refers to the stage in which students begin to identify and articulate specific properties of mathematical objects or relationships previously experienced as part of a broader image. In this interaction, students were guided to notice the relationship between the number of objects, their corresponding prices, and the algebraic representation used to calculate the total cost. This interaction reflects the Property Noticing layer because students were beginning to attend to specific properties and relationships within an algebraic representation rather than treating the situation as a whole. Students recognized that different objects were represented by different symbols and associated with different unit prices. They also recognized that the number of objects had to be multiplied by the corresponding unit price before combining the resulting costs through addition (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *C.4*).

Formalizing

In this layer, the interventions involved providing examples and asking probing and follow-up questions. Based on the Piere-Kieren framework, at this layer students can define algebraic expressions. This interaction reflects the Formalizing layer because students began to express and use mathematical relationships through formal symbolic notation (The learning activity transcripts and researcher–teacher interview transcripts are provided in Appendix *C.5*).

Observing

At this layer, students could operate on algebraic expressions; however, no data were available on teacher NL's instructional interventions at this stage. The findings show differences in the highest level of mathematical understanding achieved: students taught by teacher IL reached the Structuring layer, whereas students taught by teacher NL did not. This difference should not be interpreted as evidence that teacher interventions alone determined students' levels of understanding. In particular, the study did not establish the equivalence of students' initial mathematical abilities across the two classes, and differences in prior knowledge may have influenced the development of students' mathematical understanding. Thus, the observed differences may reflect the interaction of several factors, including students' initial cognitive abilities, the nature and responsiveness of teacher interventions, and other contextual conditions during learning. In this context, teacher IL's use of probing questions and emphasis on conceptual connections appeared to give students opportunities to reorganize and integrate their mathematical understanding. In contrast, teacher NL's instructional approach was characterized more by procedural guidance. Nevertheless, given the absence of baseline data on students' initial abilities, these observations should be interpreted cautiously and should not be taken as evidence of a causal relationship between teacher intervention and the attainment of the Structuring layer.

As shown in Table 1, the table presents the distribution of codes identified across the layers of mathematical understanding based on the NVivo analysis. The results show that the highest coding frequency occurred at the Primitive Knowing layer, with 44 coded references involving all three subjects. This indicates that the observed learning interactions contained substantial evidence of teachers' efforts to activate and elicit students' prior mathematical knowledge. The Image Having layer was coded 20 times, followed by Property Noticing with 21 coded references. Meanwhile, Image Making and Formalizing each accounted for 19 coded references. These findings indicate that the learning interactions provided evidence of teacher interventions across the different layers of students' mathematical understanding.

The Observing layer appeared six times across two subjects, indicating less frequent evidence than the preceding layers. This lower frequency may reflect the more complex understanding represented at this layer, in which students were required to use previously developed mathematical relationships to examine or interpret mathematical situations. At the same time, the teacher provided interventions to support their understanding. The Structuring layer was not identified in the NVivo coding presented in the table, while the Inventising layer also received zero codes.

The code distribution also shows differences in the number of subjects represented at each layer. Three subjects were represented at the Primitive Knowing, Image Making, Image Having, Property Noticing, and Formalizing layers, whereas only two subjects reached the Observing layer. This pattern is consistent with the qualitative analysis, which indicates that students did not demonstrate the same level of mathematical understanding. Instead, code occurrences tended to decrease as the analysis progressed toward the more advanced layers.

Based on Table 2, the interventions employed across the layers of students' mathematical understanding primarily involved questioning, including convergent, probing, and follow-up questions, as well as leading questions. Teachers also provided simplified examples and organized students into small-group discussions. These interventions can be interpreted as efforts to make abstract algebraic ideas more accessible and to give students opportunities to articulate their mathematical thinking. From the perspective of Pirie–Kieren theory, simplified examples may help students strengthen their understanding within the inner layers, such as Primitive Knowing and Image Making. At the same time, small-group discussions may give students opportunities to externalize and negotiate their mathematical ideas through interaction.

A notable finding in this study is the predominance of convergent questions in the observed classroom discourse. This finding aligns with international research showing that teacher questioning often remains teacher-directed in mathematics classrooms. For example, McCarthy et al. (2016), in a study of Grade 8 mathematics classrooms in the United States, identified several questioning strategies, including probing and follow-up, leading, checklist, and student-specific questioning, and emphasized that the nature of teachers' questions influences the opportunities available for students to articulate their mathematical thinking. Similarly, research examining teacher questioning practices has distinguished between *funneling* and *focusing* patterns. In a funneling pattern, teachers use a sequence of questions to guide students toward a predetermined answer. In contrast, a focusing pattern uses students' contributions as resources for further mathematical reasoning and encourages students to explain and reflect on their own thinking.

The predominance of convergent questions observed in the present study can therefore be understood as a form of structurally guided classroom discourse. Such questions can help check understanding, direct students' attention to relevant mathematical information, and guide them through a solution. However, when convergent questioning becomes the dominant discourse pattern, students may have fewer opportunities to formulate explanations, justify relationships, or develop mathematical ideas independently. This interpretation is consistent with research showing that closed-ended questioning and teacher-led interaction tend to elicit shorter student responses. In contrast, open-ended questions combined with more dialogic interaction provide greater opportunities for students to justify and develop their mathematical reasoning.

At the same time, the findings should not be interpreted as suggesting that convergent questions are inherently ineffective. International research on mathematics teacher questioning indicates that lower-order or more focused questions can serve important scaffolding functions, particularly when used to establish foundational understanding or support students struggling with a task. DeJarnette et al. (2020), for instance, noted that a question's contribution depends not only on its cognitive demand but also on its instructional purpose and its relationship to subsequent classroom discourse. Thus, the issue is not simply the frequency of convergent questions but how teachers sequence them with probing, follow-up, and more open-ended questions.

In the present study, probing and follow-up questions appeared to provide greater opportunities for students to clarify and elaborate their mathematical thinking, particularly when teachers responded to students' initial answers rather than immediately supplying the expected procedure. This pattern matters in relation to the Pirie–Kieren framework because movement between layers is not achieved by obtaining a correct answer alone; rather, students need opportunities to reflect on, reorganize, and extend their existing understanding. Consequently, combining convergent questioning with probing and follow-up questions may have supported students' movement through the inner and middle layers of understanding. At the same time, the limited use of more exploratory and open-ended discourse may have constrained opportunities for students to move toward more sophisticated forms of mathematical generalization.

These findings also help explain why most observed learning processes did not extend beyond the Structuring layer. The teachers' interventions generally supported students in clarifying, connecting, and organizing existing mathematical ideas. However, there was comparatively less evidence of discourse that explicitly invited students to generate conjectures, explore alternative relationships, or construct new mathematical ideas. Research on mathematical classroom discourse similarly suggests that teacher-led or funneling interactions can guide students efficiently toward normative solutions. However, it may provide fewer opportunities for students to make their own mathematical thinking visible and develop it collectively. Therefore, the absence of evidence for the Inventising layer should not be attributed solely to the teachers' questioning practices; rather, it may reflect the interaction among the nature of classroom discourse, the instructional tasks, students' existing mathematical understanding, and other contextual conditions.

Theoretically, this study contributes to the application of Pirie–Kieren theory by illustrating how naturally occurring classroom discourse can be examined in relation to students' movement across layers of mathematical understanding. The findings suggest that different forms of teacher questioning may offer different opportunities for students to reorganize and extend their understanding. Practically, while convergent questions, simplified examples, and small-group discussions remain valuable for establishing and reinforcing foundational understanding, teachers may need to complement these strategies with more probing, focusing, exploratory, and open-ended questions. Such discourse practices can give students more opportunities to explain, justify, compare, generalize, and construct mathematical relationships, potentially supporting movement toward higher layers of mathematical understanding.

Pirie & Kieren (1994) classified teachers' interventions as provocative, invocative, and validating based on their direct impact on the direction (inward or outward) of an understanding activity. However, in this study, we observed teacher-student interaction in the classroom. Interventions were carried out by the teacher at each layer of student understanding. The intervention carried out by the teacher during class learning is done by asking questions and providing simpler examples so that students can easily understand algebraic expressions. This intervention is carried out by teachers to advance their students' mathematical understanding (Wright, 2014).

In the primitive knowing, image making, image having, and property noticing layers, the teacher encourages students to think by asking questions. The types of questions asked by the teacher in the learning process are convergent questions, probing and follow-up questions, and leading questions. Convergent questions, probing and follow-up questions, and leading questions help improve students' mathematical understanding (Faradiba et al., 2019; Manyuchi, 2016; Zayyadi et al., 2019; Tofade et al., 2013; Lanya et al., 2024). In addition, the teacher intervenes by grouping students for small-group discussions (Faradiba et al., 2019; Jin et al., 2022).

In addition, one form of teacher intervention observed in this study was providing examples that were simpler than the given tasks. Within the Pirie-Kieren framework, this intervention appeared to help students revisit earlier layers of understanding before progressing to more advanced reasoning. While the present study analyzed these interactions solely through the Pirie-Kieren lens, the classroom transcripts also suggest opportunities for complementary analysis using teacher movement frameworks (Lanya et al., 2026; Ellis et al., 2019; Julian, 2006). Such frameworks could more fully characterize how teachers initiate, sustain, and redirect students' mathematical thinking during classroom interactions. Combining the Pirie-Kieren model, which focuses on the development of students' understanding, with teacher movement perspectives, which emphasize instructional actions, may offer a more comprehensive interpretation of the observed teacher-student interactions. Therefore, future studies could integrate these complementary frameworks to further examine the relationship between teacher interventions and the progression of students' mathematical understanding (Darmawan et al., 2021).

Each type of intervention the teacher provides addresses different layers of students' mathematical understanding. The types of teacher intervention identified in this study were submitting convergent questions, probing and follow-up, and leading questions. In addition, teachers provide simpler or spontaneous examples and group students in small groups to make the learning material easier to understand, especially algebra.

■ CONCLUSION

This study is limited to the understanding layers proposed by the Pirie-Kieren theory, with the participants consisting of junior high school mathematics teachers teaching algebraic expressions. Classroom observations were conducted throughout the instructional sequence until the daily assessment was administered. However, only two instructional sessions for each teacher were analyzed using the Pirie-Kieren framework. Consequently, interpret the findings as a description of the teacher interventions and the corresponding understanding layers observed during those sessions, rather than as evidence of the effectiveness or causal impact of particular interventions on students' mathematical understanding. Therefore, the results may not fully represent overall learning of algebraic expressions or generalize to broader instructional contexts. This research is limited to the theory of understanding layers proposed by Pirie-Kieren, and the subjects observed were junior high school mathematics teachers teaching algebraic expressions. Observation was conducted throughout the teaching of algebraic material up to the administration of daily assessments. However, the study analyzed only two teaching sessions per teacher using the Pirie-Kieren model of understanding. Therefore, the findings may not fully represent overall outcomes of learning algebraic expressions in a broader instructional context.

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