



Tracing Students' Integrative Thinking Development Through Adaptive Integral Worksheets: Evidence from Single-Subject Research

Ina Noviani, Niswa Nagina Lutfiah, Sulistiawati, Rina Oktaviyanthi*

Department of Mathematics Education, Universitas Serang Raya, Indonesia

ARTICLE INFO

Article History:

Received: 12 July 2026

Accepted: 24 August 2026

Published: 30 September 2026

Keywords:

Integrative thinking;
Integral problem solving;
Multiple representations;
Single-subject research;
Students' thinking stages.

ABSTRACT

This study aimed to map students' integrative thinking stages in solving integral problems presented in multiple representations. A Single Subject Research (SSR) using an A-B design was conducted involving three senior high school students categorized as high, moderate, and low ability. The students were selected purposively, and data were collected through task sheets, think-aloud protocols, and written responses across baseline and intervention phases. The intervention used adaptive integral worksheets based on four stages of integrative thinking: Articulate the Models, Examine the Models, Explore the Possibilities, and Assess the Prototypes. Data analysis combined visual plotting and qualitative interpretation. The results showed that moderate- and low-ability students improved significantly, with 100% non-overlapping data (PND), while the high-ability student showed stable performance with minor variations. Qualitative analysis revealed that high-ability students demonstrated advanced articulation but required additional cognitive challenges to sustain focus under time pressure. Meanwhile, moderate- and low-ability students showed consistent progress across stages, especially when transitioning between representations. This research concludes that adaptive worksheets effectively enhance integrative thinking and support differentiated instructional design tailored to students' representational reasoning.

To cite this article:

Noviani, I., Lutfiah, N. N., Sulistiawati, & Oktaviyanthi, R. (2026). Tracing students' integrative thinking development through adaptive integral worksheets: Evidence from single-subject research. *Pedagogy Review*, 5(3), 512-523. doi: <https://doi.org/10.61436/pedrev/v5i3.pp512-523>

This is an open access article under the CC BY SA licence (<https://creativecommons.org/licenses/by-sa/4.0>).

■ INTRODUCTION

Integral calculus is a key learning objective in the high school mathematics curriculum. Understanding integrals requires not only procedural competence in solving problems but also conceptual comprehension through various representations, such as graphs, algebraic symbols, verbal narratives, and numerical tables (Jones, 2013; Nedaei et al., 2022). In instructional practice, the ability to interrelate and switch between representations is essential for constructing a holistic understanding of integrals, both as the limit of Riemann sums and as the area under a curve. This ability reflects integrative thinking, defined as the capacity to combine diverse representational perspectives to analyze and solve problems comprehensively (Hu, 2024; Wu & Molnár, 2022). In mathematical problem solving, integrative thinking enables students to navigate complex information, evaluate representational strategies, and establish meaningful conceptual connections (Doerr & Lesh, 2003). Thus, mapping and fostering this cognitive skill is crucial, particularly when students encounter integrals presented in multi-

representational formats.

In practice, students often face various difficulties when solving problems involving multiple representations (graphical, symbolic, numerical, and verbal). In the Articulate the Models stage, many students struggle to interpret the definite integral geometrically as the area under a curve or fail to connect the symbolic representation $\int_a^b f(x)dx$ with the corresponding graph of $f(x)$. Some misinterpret the integral as a simple algebraic operation rather than an accumulation of area. In the Examine the Models stage, students often fail to compare analytical methods (applying integration rules) with simpler numerical approaches, such as the trapezoidal rule, or to assess their limitations. During the Explore the Possibilities stage, students encounter difficulties with applied problems, such as calculating distance from velocity graphs, volumes of solids of revolution, or areas between two curves. Finally, in the Assess the Prototypes stage, students often do not reflect on or verify their results, such as questioning whether a calculated area can logically be negative. When students do not understand

these four stages well, their integrative thinking is hindered, and they fail to use integrals effectively to model and solve real-world problems.

The suboptimal development of students' integrative thinking in solving integral-based multi-representational problems may stem from a mismatch between instructional media and students' cognitive needs (Danday, 2023). Therefore, adaptive learning media are needed to systematically and contextually facilitate connections among verbal, symbolic, and visual representations (Joo et al., 2021; Turčáni et al., 2024). One promising medium is the adaptive integral worksheet, designed to provide scaffolding that aligns with students' developmental stages and individual abilities. Noviani et al. (2024) found that such worksheets can support deductive and analytical thinking in constructing concepts of area, volume, and integral accumulation across different representations. In this context, the current study adopts a Single Subject Research (SSR) approach to map students' integrative thinking stages based on their interactions with the worksheet. This mapping process is expected to inform the development of more adaptive and effective instructional strategies that accommodate diverse cognitive needs.

Although prior research has explored the development of integral worksheets, it remains relatively limited compared to topics like trigonometry or geometry. Sinaga et al. (2022) highlighted the importance of visual-representation-based scaffolding in integral worksheets for high school students. Borji et al. (2024) developed contextualized worksheets that connect area and volume concepts with real-life phenomena. Oktaviyanthi and Agus (2023) demonstrated that adaptive worksheet structures enhance the deductive reasoning skills of prospective teachers when modeling accumulation functions. While many studies have validated the practicality and effectiveness of integral worksheets, research mapping individual students' cognitive processes remains scarce. The SSR approach is particularly well suited to exploring the dynamics of integrative thinking, including error patterns, representational strategies, and transitions between cognitive stages (Becker, 2023; Hochberg et al., 2020; Levin & Ferron, 2021). Through think-aloud protocols and visual representations collected during problem solving, SSR enables comprehensive and contextual mapping of integrative thinking stages.

Given the gap between ideal needs for developing integrative thinking and instructional practices that often overlook

students' representational difficulties and conceptual transitions, this study evaluates the effectiveness of adaptive integral worksheets in mapping students' integrative thinking stages. The worksheet is designed to stimulate four cognitive actions: Articulate the Models, Examine the Models, Explore the Possibilities, and Assess the Prototypes, in order to reveal how students engage with multi-representational tasks involving key integral concepts such as area under a curve, accumulation functions, and substitution strategies. This study aspires to both optimize students' mastery of integral concepts through adaptive media and provide a diagnostic framework for educators to identify thinking patterns, misconceptions, and cognitive development in the context of integrals. Based on the identified research gap, this study addresses two research questions. RQ1 explores how students' integrative thinking stages develop during the implementation of adaptive integral worksheets, whereas RQ2 examines how these developmental patterns differ among students with high, moderate, and low initial mathematical ability.

■ METHOD

Research Design

This study aimed to evaluate how the intervention using Adaptive Integral Worksheets influences the development of high school students' integrative thinking stages in solving multi-representational mathematical problems. A quantitative analysis was conducted by tracking changes in three students representing high, moderate, and low ability levels. Although interventions are typically conducted in larger groups, this study employed a Single Subject Research (SSR) design, focusing on in-depth case studies. SSR is a single-case experimental design that examines changes in individuals through repeated observations over a specific period (Catagnus et al., 2023). This approach enables the evaluation of intervention effects using a small number of subjects, with each participant serving as their own control (Yenioğlu et al., 2022; Zuidersma et al., 2020). SSR systematically compares baseline and intervention phases to assess intervention-related changes (Lanovaz & Primiani, 2023). In this study, the Adaptive Integral Worksheet served as the independent variable, while students' integrative thinking performance functioned as the dependent variable.

The research procedure consisted of four sequential stages: participant selection, baseline observation, implementation of the Adaptive Integral Worksheet during the intervention

phase, and analysis of changes in students' integrative thinking. Quantitative performance data and qualitative evidence from written responses and think-aloud protocols were collected across the observation sessions and integrated during data analysis.

Participants

Three senior high school students were selected using purposive sampling to represent high, moderate, and low ability groups. Participant selection was based on three criteria: (1) an initial assessment of integral concepts, (2) mathematics report card grades, and (3) mathematics teacher recommendations. Students were categorized as high ability (≥ 85), moderate ability (70–84), and low ability (< 70) based on the combined initial assessment and mathematics report card scores. Teacher recommendations were subsequently used to confirm that the selected participants appropriately represented each ability group. Before data collection, all participants had completed instruction on basic integral concepts according to the school curriculum. The three selected students represented different levels of mathematical ability, allowing the study to examine variations in the development of integrative thinking during the intervention.

Ethical Consideration

Prior to data collection, permission to conduct the study was obtained from the participating school. The Research Ethics Committee, Institute for Research and Community Service (LPPM), Universitas Serang Raya, Indonesia granted ethical approval (Approval No. 101/01.03/LPPM/E.08/II/2025). Students and their parents or legal guardians received written information regarding the study objectives, research procedures, voluntary participation, confidentiality of personal information, and the use of think-aloud recordings for research purposes. Written informed consent was obtained from all participants and their parents or legal guardians before the study commenced. To protect participants' privacy, pseudonyms (S1, S2, and S3) were used throughout the study.

Intervention and Research Procedure

The study was conducted in two consecutive phases: Baseline (A) and Intervention (B). During the Baseline phase, students learned using the standard senior high school mathematics textbook and completed multi-representational integral tasks without additional instructional support. This phase was

intended to establish each student's initial profile of integrative thinking. Subsequently, during the Intervention phase, students learned using the adaptive integral worksheet, which was designed to facilitate the four stages of integrative thinking: Articulate the Models, Examine the Models, Explore the Possibilities, and Assess the Prototypes (Noviani et al., 2024). Each phase consisted of five 60-minute sessions. Sessions were conducted twice a week, with approximately 2–3 days between consecutive observations, and the intervention phase began immediately after completion of the baseline phase. Thus, the ten observation sessions were completed over approximately five weeks using a consistent schedule across phases. The relatively short, consistent observation period limited temporal variation across measurements, although maturation cannot be ruled out as an alternative explanation within an A–B single-subject design (Horner et al., 2005). Students completed one worksheet during each intervention session while verbalizing their thinking through the think-aloud protocol. Written responses and verbal protocols were collected throughout the intervention.

Instruments

Four instruments were used to collect the research data: the adaptive integral worksheet, a multi-representational integral problem-solving test, a think-aloud protocol, and students' written responses. The problem-solving test consisted of two open-ended problems designed to assess students' integrative thinking while solving integral problems. Each problem required students to interpret mathematical situations, compare alternative solution strategies, connect symbolic and graphical representations, and evaluate the appropriateness of their solutions. Rather than focusing solely on the correctness of the final answer, students' responses were evaluated using an analytic rubric covering the four stages of integrative thinking. Table 1 summarizes the measured indicators and scoring indicators for each stage.

Each scoring indicator was rated on a 0–5 analytic scale based on the completeness and quality of the reasoning demonstrated in students' responses. A score of 0 indicated no evidence of the targeted reasoning; 1 indicated very limited evidence; 2 indicated partial but incomplete evidence; 3 indicated adequate evidence with some limitations; 4 indicated substantial and mostly complete evidence; and 5 indicated complete and conceptually appropriate evidence. Partial scores between these anchor levels were permitted when a

Table 1. Integrative thinking indicators and scoring structure

Integrative Thinking Stage	Measured Indicator	Number of Scoring Indicators	Score Range
Articulate the Models	Identifying and interpreting multiple mathematical representations	4	0–5
Examine the Models	Comparing alternative mathematical solution strategies	4	0–5
Explore the Possibilities	Exploring multiple representations and possible solution pathways	4	0–5
Assess the Prototypes	Evaluating, justifying, and reflecting on the appropriateness of solutions	4	0–5
Total		16	0–80

response demonstrated an intermediate level of performance. This scoring procedure allowed gradual differences in students' reasoning to be represented rather than restricting the assessment to discrete performance categories. Across the 16 scoring indicators, the maximum possible raw score was 80. The scores were summed and converted to a standardized score ranging from 0 to 100.

The standardized score was reported to one decimal place and served as the performance measure for each observation session during both the baseline and intervention phases. Prior to implementation, the instrument was evaluated by five mathematics education experts using a five-point relevance scale. Content validity was examined using Aiken's *V*, with item-level coefficients ranging from 0.75 to 0.95 and an overall coefficient of 0.84, indicating good content validity (Penfield & Giacobbi, 2004; Spoto et al., 2023). Minor revisions were made based on the experts' qualitative feedback. The revised instrument was then pilot-tested with 15 students not included in the main study. Internal consistency of the 16 scoring indicators was examined using Cronbach's α , yielding a coefficient of 0.914, indicating high internal consistency.

Qualitative data were collected through the think-aloud protocol and students' written responses. The think-aloud recordings captured students' verbal reasoning during problem solving, while the written responses documented their representational strategies and solution processes. These qualitative data complemented the quantitative scores and were triangulated during the data analysis.

Data Analysis

The study employed both quantitative and qualitative analyses. Quantitative data were

analyzed using standard visual analysis procedures for Single Subject Research (SSR) (Kemp & Fisher, 2023). The analysis examined level, trend direction, trend stability, immediacy of effect, data overlap, consistency of data patterns, within-phase analysis, between-phase analysis, and the Percentage of Non-overlapping Data (PND). Trend estimation was performed using the split-middle method, whereas stability analysis followed the split-half procedure recommended by Levin and Ferron (2021). The effectiveness of the adaptive integral worksheet was evaluated by comparing changes in level, trend, stability, and PND values between the baseline and intervention phases.

Qualitative data from the think-aloud recordings and students' written responses were analyzed using qualitative content analysis. The recordings were first transcribed verbatim and matched with the corresponding written responses from each observation session. The transcripts were then reduced by removing verbal expressions unrelated to mathematical reasoning, after which meaningful segments were coded according to the four stages of integrative thinking: Articulate the Models, Examine the Models, Explore the Possibilities, and Assess the Prototypes. The coded data were subsequently interpreted to identify changes in students' cognitive patterns across the baseline and intervention phases.

To enhance the credibility of the findings, qualitative interpretations were triangulated by comparing students' verbal explanations, written responses, and SSR performance from the same observation sessions. Coding and interpretation were reviewed through researcher discussion until consensus was achieved, ensuring that the identified cognitive patterns were consistently supported by multiple sources of evidence.

■ RESULT AND DISCUSSION

Within-Phase Analysis

The within-phase analysis examined the performance trends of S1, S2, and S3 during the baseline and intervention phases. Stable data patterns within each condition matter because changes are easier to detect under stable conditions (Dowdy & Jessel, 2024). Each participant was observed across five sessions in each phase. Figure 1 presents the session-by-session scores and corresponding stability boundaries for the baseline (A) and intervention (B) phases.

The first row of Figure 1 shows the score distribution across the baseline and intervention phases, while the second row presents the corresponding stability bands. Data stability was determined by the percentage of observations falling within $\pm 15\%$ of the phase

median, with approximately 85% used as the criterion for stable performance (Kazdin, 2021). Trend direction and trend stability were further examined using the split-middle procedure. Table 2 summarizes the within-phase characteristics for each participant and phase, including mean, median, range, data stability, trend direction, trend stability, and trend pattern.

Table 2 shows that mean performance increased from the baseline to the intervention phase for all three subjects. Data stability remained at 80% across conditions, while trend stability ranged from 80% to 100%. All participants displayed accelerating, stable, and single trend patterns in both phases. These results indicate that the observed changes occurred within generally consistent data patterns rather than through isolated

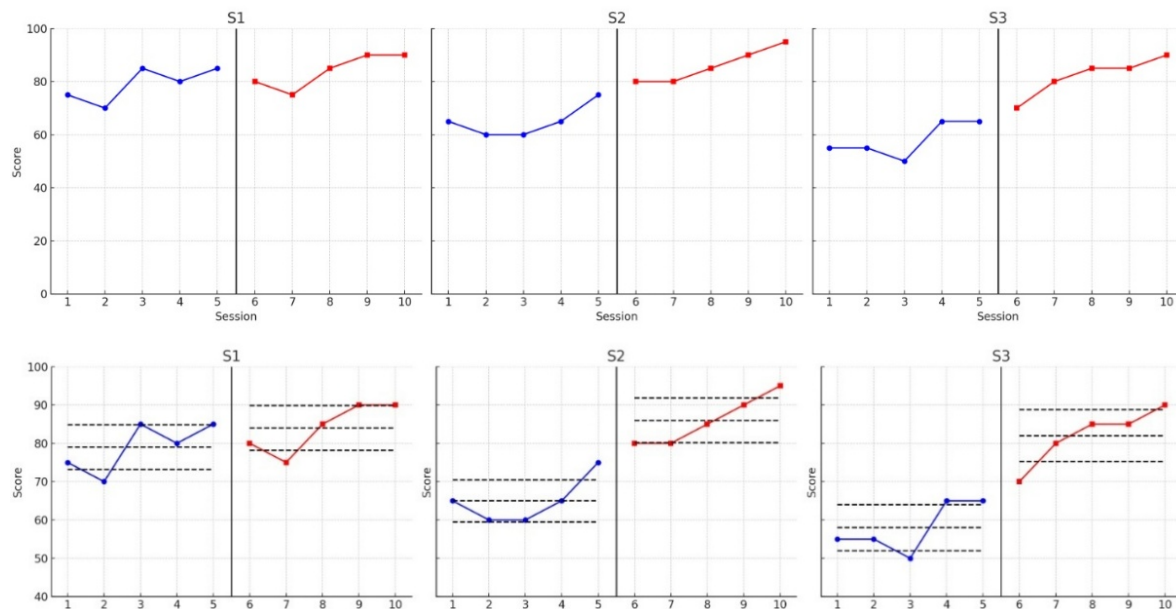


Figure 1. Performance trends and stability bounds for S1, S2, and S3

Table 2. Summary of within-phase SSR analysis

Subject	Phase	Mean	Median	Range	Data Stability	Trend Direction	Trend Stability	Trend Pattern
S1	Baseline (A)	79.24	80.1	15.2	80%	Acceleration	Stable (80%)	Single
	Intervention (B)	84.28	85.4	15.2	80%	Acceleration	Stable (80%)	Single
S2	Baseline (A)	65.32	65.2	15.4	80%	Acceleration	Stable (100%)	Single
	Intervention (B)	86.24	85.3	15.0	80%	Acceleration	Stable (100%)	Single
S3	Baseline (A)	58.24	55.5	15.4	80%	Acceleration	Stable (80%)	Single
	Intervention (B)	82.30	85.5	20.5	80%	Acceleration	Stable (80%)	Single

Table 3. Relative and absolute level changes for each condition (S1, S2, S3)

Aspect of Level Change		Subject S1		Subject S2		Subject S3	
		Phase A	Phase B	Phase A	Phase B	Phase A	Phase B
Relative Change	Median of 1 st half (Q1)	72.75	77.75	60.3	80.25	52.65	75
	Median of 2 nd half (Q2)	85.3	90.25	70.4	92.7	65.2	88
	Relative Level Change	+12.55	+12.5	+10.1	+12.45	+12.55	+13
Absolute Change	First Value	75.3	80.2	65.2	80	55.3	70
	Last Value	85.2	90.5	75.5	95	65.4	90.5
	Absolute Level Change	+9.9	+10.3	+10.3	+15	+10.1	+20.5

fluctuations. To further examine the magnitude of change within each condition, relative and absolute level changes were analyzed, as presented in Table 3.

From Table 3, it can be concluded that, in general, levels changed relatively consistently across all research subjects during both the baseline and intervention phases, with changes ranging from +9 to +20. Subsequently, the trend of data point distributions was identified to determine whether the trends were monotonically increasing (acceleration trend) or decreasing (deceleration trend). Using the split-middle technique, the trend within each phase and the stability region for each subject were estimated, as illustrated in Figure 2.

Figure 2 visually confirms the accelerating trend directions and stability characteristics summarized in Table 2. Taken together, the within-phase analyses indicate consistent positive performance patterns across both conditions, with larger changes particularly evident for S2 and S3 during the intervention phase.

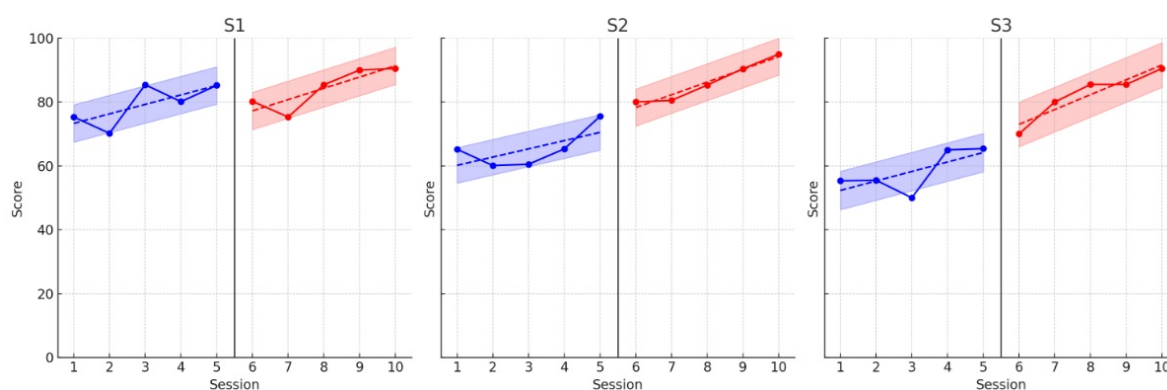
Between-Phase Analysis

The objective of the between-phase analysis is to observe behavioral differences in the research subjects between the baseline and

intervention phases (Dowdy & Jessel, 2024; Kazdin, 2021). These behavioral differences are traced through the development of integrative thinking skills, which are expected to improve significantly from the baseline to the intervention phase. Therefore, students' integrative thinking skills in solving multi-representational mathematical problems served as the dependent variable, with changes across phases being analyzed.

Differences in students' integrative thinking between the baseline and intervention phases were examined by comparing changes in performance level together with the direction and stability of trends. Figure 3 shows a clear increase in mean performance for all participants following the intervention. While S1 showed modest improvement because of an already high baseline, S2 and S3 showed substantially larger gains, indicating greater responsiveness to the adaptive integral worksheet. These findings provide further evidence that the intervention contributed to sustained improvements in students' integrative thinking across repeated observation sessions.

Positive level changes indicate improvements in integrative thinking from the baseline to the intervention phase. Subsequently, the analysis examined whether

**Figure 2.** Trend estimations and stability regions for each data phase of S1, S2, and S3

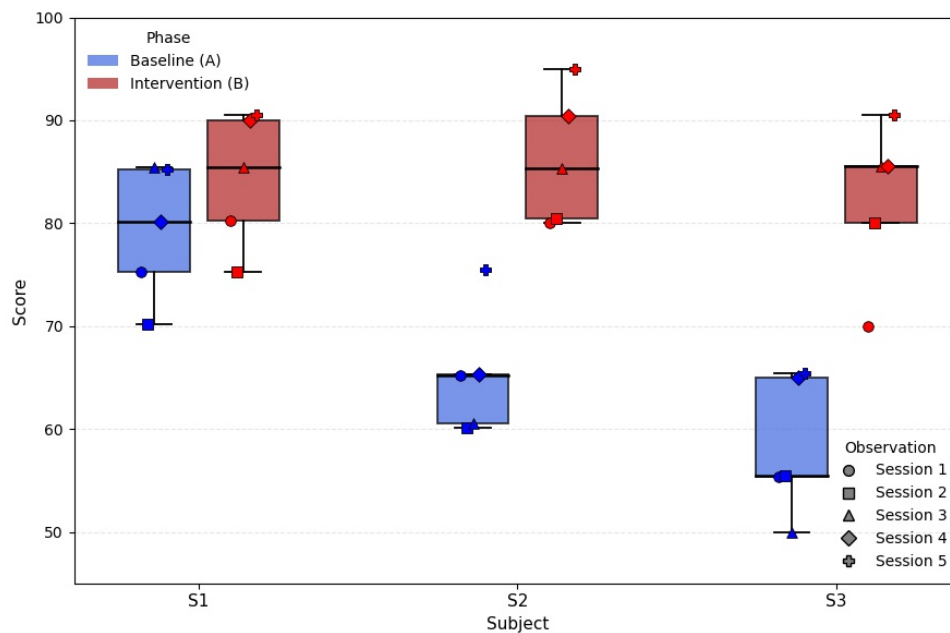


Figure 3. Baseline and intervention performance across participants

significant changes in trend direction and trend stability occurred between phases. This information is presented in Table 4.

In addition to changes in level and trends, another aspect analyzed was the Percentage of Non-overlapping Data (PND) between baseline and intervention phases. The percentage of overlap indicates how much of the intervention phase data still overlaps with the baseline data. The smaller the percentage of overlap, the clearer the difference between the two phases. The PND for each research subject is summarized in Table 5.

As shown in Table 5, S2 and S3 showed complete non-overlap between the baseline and intervention phases, with all five intervention scores exceeding their respective baseline

maxima (PND = 100%). In contrast, two of S1's five intervention scores exceeded the highest baseline score of 85.4, resulting in a PND of 40%. This indicates less separation between the two phases for S1 than for S2 and S3. However, S1's mean score increased from 79.24 during baseline to 84.28 during intervention. Therefore, the PND value was interpreted together with changes in level and the overall data pattern rather than as a standalone indicator of intervention effectiveness. Visual illustrations of the PND results for each subject are shown in Figure 4.

The analysis of SSR performance scores from the baseline (A) and intervention (B) phases provides a clear picture of the development of students' integrative thinking

Table 4. Trend and stability changes between phases

Subject	Trend Direction A	Trend Direction B	Stability A	Stability B	Trend Pattern Across Phases
S1	Acceleration	Acceleration	Stable	Stable	Consistent
S2	Acceleration	Acceleration	Stable	Stable	Consistent
S3	Acceleration	Acceleration	Stable	Stable	Consistent

Table 5. Percentage of non-overlapping data (PND) across subjects

Subject	Highest Scores Phase A	Phases B Data	Phase B data > Highest A	PND (%)
S1	85.4	80.2; 75.3; 85.4; 90; 90.5	90; 90.5 → 2 out of 5	$\frac{2}{5} \times 100\% = 40\%$
S2	75.5	80; 80.5; 85.3; 90.4; 95	All → 5 out of 5	$\frac{5}{5} \times 100\% = 100\%$
S3	65.4	70; 80; 85.5; 85.5; 90.5	All → 5 out of 5	$\frac{5}{5} \times 100\% = 100\%$

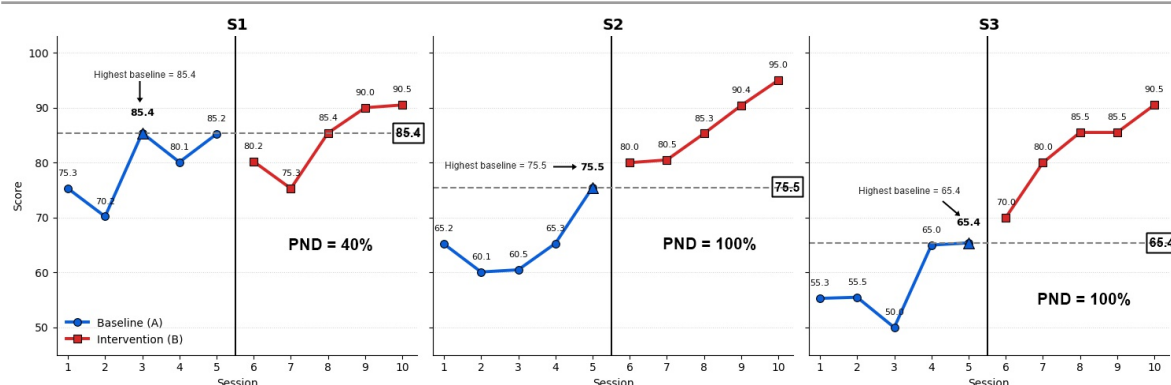


Figure 4. PND analysis with baseline thresholds

abilities in solving multi-representational mathematical problems. Score stability, with trend stability ranging from 80% to 100% and positive acceleration trends, indicates that students consistently internalized and applied the four stages of integrative thinking: Articulate, Examine, Explore, and Assess. During the baseline phase, SSR scores were generally low and fluctuated, reflecting students' limited ability to articulate mathematical models and systematically explore solutions. However, during the intervention phase, SSR scores increased significantly, indicating substantial progress in executing the stages of integrative thinking. This improvement was marked by stable scores and upward trends, reflecting progressive and sustained development of students' integrative thinking skills throughout the instructional

process.

To complement the quantitative findings, qualitative evidence from the think-aloud protocols and students' written responses was synthesized into a concept map illustrating the cognitive transitions of S1, S2, and S3 across the four stages of integrative thinking. The concept map highlights similarities and differences in their reasoning pathways, while the subsequent narrative provides representative excerpts supporting these transitions.

The concept map shows that although all three participants progressed through the same four stages of integrative thinking, their reasoning pathways differed in depth and progression. S1 demonstrated an advanced pathway, S2 showed gradual progression, and S3 exhibited a scaffolded progression from

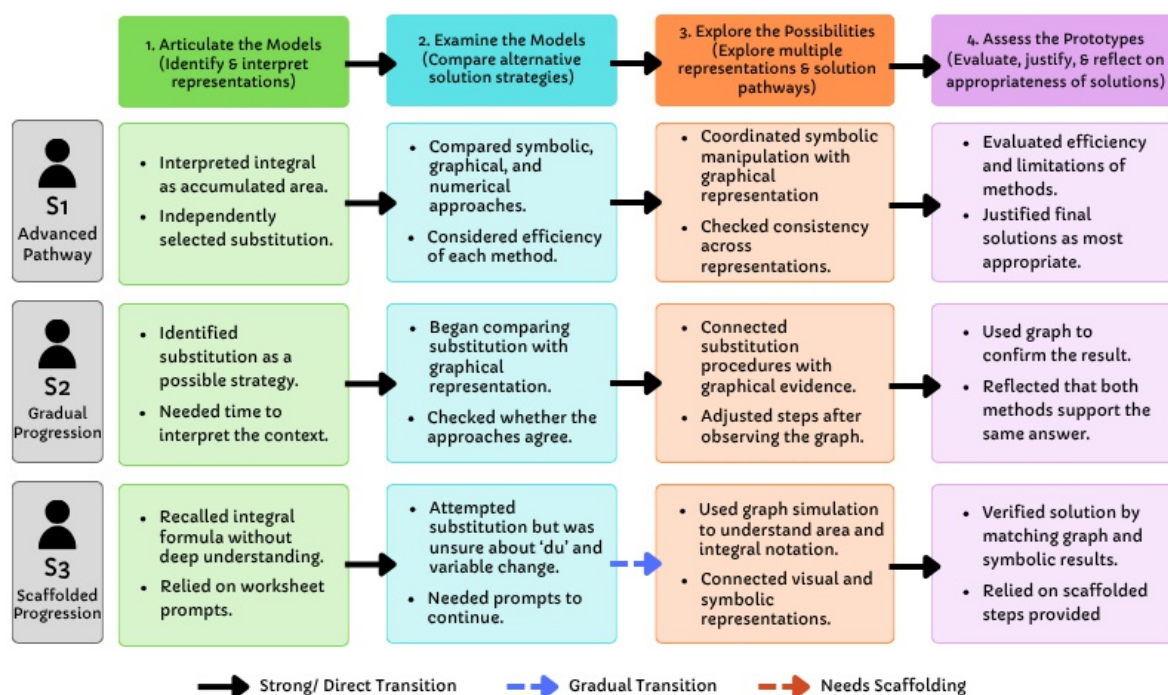


Figure 5. Cognitive transitions across the four stages of integrative thinking

limited conceptual understanding toward coordination of graphical and symbolic representations. S1 demonstrated the most advanced cognitive pathway across the four stages of integrative thinking. During the baseline phase, S1 already conceptualized the indefinite integral as an “accumulated area”, independently selected substitution, compared multiple solution strategies, and evaluated their efficiency. These characteristics were reflected in consistently high SSR scores (75.3–85.2), which further increased to 90.5 during the intervention phase, indicating refinement of an already well-established reasoning process (Carvalho & Santos, 2022). Representative evidence from the written response showed: “*I tried using substitution $u=x^2$, so that $du=2x dx$, which simplifies the calculation.*” During the think-aloud session, S1 further reflected: “*Numerical methods are slower but useful when the function is difficult to differentiate.*” These responses indicate that S1 had progressed beyond procedural execution toward reflective evaluation of alternative solution strategies.

S2 followed a gradual cognitive progression from procedural recall to representational integration. Initially, S2 relied on memorized procedures and expressed uncertainty, as reflected in: “*I used substitution, but I'm not confident.*” During the intervention, S2 began comparing symbolic and graphical representations and articulated more coherent reasoning: “*I compared the result of this substitution with the graph, and it made sense because the area increased as the graph rose.*” This transition was accompanied by an increase in SSR performance from 65.2–75.5 during the baseline phase to consistently higher scores during the intervention, resulting in a PND of 100% (Blair & Ku, 2022). S3 exhibited a scaffolded progression from limited conceptual understanding toward representational coordination. Initially, S3 demonstrated limited conceptual understanding, stating: “*I just wrote the formula, but I don't understand it.*” With support from the Adaptive Integral Worksheet and graph simulation, S3 gradually connected graphical representations with integral notation, explaining: “*Seeing the graph helped me understand the area shape a bit, so I tried to match it with the integral formula.*” This progression corresponded with an increase in SSR scores from 55.3–65.4 during the baseline phase to 90.5 during the intervention, yielding a PND of 100% and indicating substantial development in integrative thinking (Li & Zhan, 2022).

Overall, the increase in SSR scores, supported by students' written responses and verbal protocols, indicates positive changes in integrative thinking performance during the intervention. S2 and S3 showed complete non-overlap between phases, with PND values of 100%, indicating clear separation between their baseline and intervention performances (Kocaöz & Yalçın, 2022). S1 showed a different pattern. Although S1's mean score increased from 79.24 during baseline to 84.28 during intervention, only two of the five intervention scores exceeded the highest baseline score of 85.4, resulting in a PND of 40%. This lower PND indicates less separation between baseline and intervention scores for S1 than for S2 and S3. This pattern should be considered in relation to the PND calculation, which uses the most extreme baseline value as the comparison threshold; intervention scores that remain below or equal to this threshold are treated as overlapping data (Scruggs & Mastropieri, 1998; Parker et al., 2009). Because the available data do not provide item-level evidence that can distinguish an instrument ceiling from a limited intervention effect, no causal explanation is assigned to S1's pattern. Instead, S1's PND is interpreted alongside the observed increase in mean performance and the within-phase data pattern.

This finding aligns with Csikszentmihalyi's optimal challenge framework (Donaldson et al., 2025; Heutte et al., 2021), which emphasizes aligning task difficulty with students' abilities to foster cognitive flow that supports learning. The differentiated instruction approach similarly suggests that high-achieving students benefit from adaptive challenges, such as open investigations, context-based problem solving, or strategic exploration that demands deep reflection (Mofield, 2022; Roberts & Inman, 2023). Accordingly, S1's low PND signals that future interventions should account for variability in students' initial cognitive levels (Li et al., 2025). In the context of mapping integrative thinking stages, a *tiered intervention model* may serve as a promising approach to progressively and personally adjust cognitive stimuli (Farmer et al., 2021; Gutierrez De Blume et al., 2023). Future research should develop layered instructional models that maintain integrative thinking consistency, especially in more open-ended, complex, and authentic problem contexts.

The A–B design allowed repeated observation of changes between baseline and intervention phases; however, the absence of experimental replication limits causal

interpretation. Although the observation schedule was relatively short and consistent, alternative explanations such as maturation or history cannot be completely excluded.

■ CONCLUSION

The findings of this study demonstrate that mapping students' integrative thinking stages through SSR scores, think-aloud protocols, and students' written responses provides a comprehensive understanding of students' cognitive development in solving multi-representational mathematical problems. The four stages of integrative thinking, namely Articulate the Models, Examine the Models, Explore the Possibilities, and Assess the Prototypes, were consistently identified through both qualitative and quantitative evidence. The increase in SSR scores from the baseline to the intervention phase, particularly for Subjects S2 and S3, indicates progressive development of integrative thinking following the implementation of the Adaptive Integral Worksheet. Although the same intervention was implemented for all participants, the students demonstrated different learning trajectories according to their initial levels of integrative thinking. These findings suggest that students with different cognitive starting points may respond differently to the same instructional support. Therefore, mapping integrative thinking stages can serve as a valuable diagnostic approach for understanding students' reasoning development and may inform future instructional interventions that better respond to students' individual learning needs.

■ ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Directorate of Learning and Student Affairs (Belmawa), Ministry of Education, Culture, Research, and Technology, Indonesia, for recognizing this article through the 2025 PKM-AI Outputs.

■ DISCLOSURE OF GENERATIVE AI AND AI-ASSISTED TECHNOLOGIES

While writing this manuscript, the authors used ChatGPT (OpenAI) and Grammarly Pro to assist with language refinement, grammar improvement, and proofreading. The authors carefully reviewed and edited all AI-assisted outputs and assume full responsibility for the accuracy, integrity, and content of the published article.

■ AUTHOR CONTRIBUTIONS

IN: Investigation, Writing – Original Draft;>NNL: Data Collection, Visualization; S:

Validation, Resources; RO: Conceptualization, Methodology, Formal Analysis, Supervision; RJR: Validation, Writing – Review & Editing. All authors have read and approved the final version of the manuscript.

■ REFERENCES

- Becker, A. (2023). The benefits of single-subject research designs and multi-methodological approaches for neuroscience research. *Frontiers in Human Neuroscience*, 17, 1190412. <https://doi.org/10.3389/FNHUM.2023.1190412>
- Blair, C., & Ku, S. (2022). A Hierarchical Integrated Model of Self-Regulation. *Frontiers in Psychology*, 13, 725828. <https://doi.org/10.3389/FPSYG.2022.725828>
- Borji, V., Martínez-Planell, R., & Trigueros, M. (2024). Students' Understanding of Riemann Sums and Double Integrals: The Case of Task Design in APOS Theory. *International Journal of Research in Undergraduate Mathematics Education*, 1–29. <https://doi.org/10.1007/S40753-024-00250-6>
- Carvalho, A. R., & Santos, C. (2022). Developing peer mentors' collaborative and metacognitive skills with a technology-enhanced peer learning program. *Computers and Education Open*, 3, 100070. <https://doi.org/10.1016/J.CAEO.2021.100070>
- Catagnus, R. M., Garcia, Y. A., & Zhang, D. X. (2023). Single-Subject Design Studies of Mindfulness Interventions for Behavior Change: A Scoping Review and Behavioral Conceptualization. *Advances in Neurodevelopmental Disorders*, 7(2), 133–164. <https://doi.org/10.1007/S41252-022-00294-2>
- Danday, B. A. (2023). *Fostering Critical Thinking Among Pre-service Teachers Through a Multiple Representation-Based Collaborative Pedagogical Approach*. 191–217. https://doi.org/10.1007/978-3-031-15959-6_11
- Doerr, H. M. ., & Lesh, R. A. . (2003). *Foundations of a Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving*. 3–33. <https://doi.org/10.4324/9781410607713-2>
- Donaldson, S. I. ., Dubin, Matt., & Csikszentmihalyi, Mihaly. (2025). *Flow 2.0 : optimal experience in a complex world : honoring Mihaly Csikszentmihalyi's legacy*. 127.
- Dowdy, A., & Jessel, J. (2024). Single Case Research Designs. *Research Methods in*

- Special Education*, 165–198. <https://doi.org/10.4324/9781003526315-10>
- Farmer, T. W., Bierman, K. L., Hall, C. M., Brooks, D. S., & Lee, D. L. (2021). Tiered Systems of Adaptive Supports and the Individualization of Intervention: Merging Developmental Cascades and Correlated Constraints Perspectives. *Journal of Emotional and Behavioral Disorders*, 29(1), 3–13. <https://doi.org/10.1177/1063426620957651>
- Gutierrez De Blume, P., Lin, H.-C. K., Drigas, A., Mitsea, E., & Skianis, C. (2023). Meta-Learning: A Nine-Layer Model Based on Metacognition and Smart Technologies. *Sustainability* 2023, Vol. 15, Page 1668, 15(2), 1668. <https://doi.org/10.3390/SU15021668>
- Heutte, J., Fenouillet, F., Martin-Krumm, C., Gute, G., Raes, A., Gute, D., Bachelet, R., & Csikszentmihalyi, M. (2021). Optimal Experience in Adult Learning: Conception and Validation of the Flow in Education Scale (EduFlow-2). *Frontiers in Psychology*, 12, 828027. <https://doi.org/10.3389/FPSYG.2021.828027>
- Hochberg, K., Becker, S., Louis, M., Klein, P., & Kuhn, J. (2020). Using Smartphones as Experimental Tools—a Follow-up: Cognitive Effects by Video Analysis and Reduction of Cognitive Load by Multiple Representations. *Journal of Science Education and Technology*, 29(2), 303–317. <https://doi.org/10.1007/S10956-020-09816-W>
- Hu, L. (2024). Programming and 21st century skill development in K-12 schools: A multidimensional meta-analysis. *Journal of Computer Assisted Learning*, 40(2), 610–636. <https://doi.org/10.1111/JCAL.12904>
- Jones, S. R. (2013). Understanding the integral: Students' symbolic forms. *The Journal of Mathematical Behavior*, 32(2), 122–141. <https://doi.org/10.1016/J.JMATHB.2012.12.004>
- Joo, H., Park, J., & Kim, D. (2021). Visual representation fidelity and self-explanation prompts in multi-representational adaptive learning. *Journal of Computer Assisted Learning*, 37(4), 1091–1106. <https://doi.org/10.1111/JCAL.12548>
- Kazdin, A. E. (2021). Single-case experimental designs: Characteristics, changes, and challenges. *Journal of the Experimental Analysis of Behavior*, 115(1), 56–85. <https://doi.org/10.1002/JEAB.638>
- Kemp, A. H., & Fisher, Z. (2023). Application of Single-Case Research Designs in Undergraduate Student Reports: An Example From Wellbeing Science. *Teaching of Psychology*, 50(1), 86–92. <https://doi.org/10.1177/00986283211029929>
- Kocaöz, O. E., & Yalçın, G. (2022). The effectiveness of Gagne's model in concept teaching for a student with intellectual disability. *Journal of Pedagogical Research*, 6(4), 101–110. <https://doi.org/10.33902/JPR.202216895>
- Lanovaz, M. J., & Primiani, R. (2023). Waiting for baseline stability in single-case designs: Is it worth the time and effort? *Behavior Research Methods*, 55(2), 843–854. <https://doi.org/10.3758/S13428-022-01858-9>
- Levin, J. R., & Ferron, J. M. (2021). Different randomized multiple-baseline models for different situations: A practical guide for single-case intervention researchers. *Journal of School Psychology*, 86, 169–177. <https://doi.org/10.1016/J.JSP.2021.03.003>
- Li, T., & Zhan, Z. (2022). A Systematic Review on Design Thinking Integrated Learning in K-12 Education. *Applied Sciences* 2022, Vol. 12, Page 8077, 12(16), 8077. <https://doi.org/10.3390/APP12168077>
- Li, Y., Li, Y., He, X., Fang, J., Zhou, C. Y., & Liu, C. (2025). Learner's cognitive state recognition based on multimodal physiological signal fusion. *Applied Intelligence*, 55(2), 1–16. <https://doi.org/10.1007/S10489-024-05958-1>
- Mofield, E. L. (2022). Vertical Differentiation for Gifted, Advanced, and High-Potential Students: 25 Strategies to Stretch Student Thinking. *Vertical Differentiation for Gifted, Advanced, and High-Potential Students: 25 Strategies to Stretch Student Thinking*, 1–182. <https://doi.org/10.4324/9781003293286>
- Nedaei, M., Radmehr, F., & Drake, M. (2022). Exploring undergraduate engineering students' mathematical problem-posing: the case of integral-area relationships in integral calculus. *Mathematical Thinking and Learning*, 24(2), 149–175. <https://doi.org/10.1080/10986065.2020.1858516>
- Noviani, I., Lutfiah, N. N., Sulistiawati, & Oktaviyanthi, R. (2024). *Pengembangan lembar kerja integral adaptif untuk integrative thinking* (Research Report). Universitas Serang Raya.
- Oktaviyanthi, R., & Agus, R. N. (2023). Evaluating graphing quadratic worksheet on visual thinking classification: A confirmatory analysis. *Infinity Journal*, 12(2), 207–224. <https://doi.org/10.22460/>

- INFINITY.V12I2.P207-224
- Penfield, R. D., & Giacobbi, Jr., P. R., Jr. (2004). Applying a score confidence interval to Aiken's item content-relevance index. *Measurement in physical education and exercise science*, 8(4), 213-225. https://doi.org/10.1207/s15327841mpee0804_3
- Roberts, J. L., & Inman, T. F. (2023). Strategies for Differentiating Instruction : Best Practices for the Classroom. *Strategies for Differentiating Instruction*. <https://doi.org/10.4324/9781003330561>
- Sinaga, P., Setiawan, W., & Liana, M. (2022). The impact of electronic interactive teaching materials (EITMs) in e-learning on junior high school students' critical thinking skills. *Thinking Skills and Creativity*, 46, 101066. <https://doi.org/10.1016/J.TSC.2022.101066>
- Spoto, A., Nucci, M., Prunetti, E., & Vicovaro, M. (2025). Improving content validity evaluation of assessment instruments through formal content validity analysis. *Psychological methods*, 30(2), 203. <https://psycnet.apa.org/doi/10.1037/met0000545>
- Turčáni, M., Balogh, Z., & Kohútek, M. (2024). Evaluating computer science students' reading comprehension of educational multimedia-enhanced text using scalable eye-tracking methodology. *Smart Learning Environments*, 11(1), 1–28. <https://doi.org/10.1186/S40561-024-00318-5>
- Wu, H., & Molnár, G. (2022). Analysing Complex Problem-Solving Strategies from a Cognitive Perspective: The Role of Thinking Skills. *Journal of Intelligence*, 2022, Vol. 10, Page 46, 10(3), 46. <https://doi.org/10.3390/JINTELLIGENCE10030046>
- Yenioğlu, S., Melekoğlu, M. A., & Yılmaz Yenioğlu, B. (2022). A single-subject case study of twice exceptionality. *Gifted Education International*, 38(2), 256-272. <https://doi.org/10.1177/02614294211064772>
- Zuidersma, M., Riese, H., Snippe, E., Booij, S. H., Wichers, M., & Bos, E. H. (2020). Single-subject research in psychiatry: Facts and fictions. *Frontiers in Psychiatry*, 11, 539777. <https://doi.org/10.3389/fpsy.2020.539777>